Little strings and T-duality

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Talk based on [JK., Seok Kim, Kimyeong Lee] in progress.

Outline

- 6d N=(1,1) Little strings
  - NS5-branes in IIB string theory in 10d flat space
  - Supersymmetric index of 6d N=(1,1) Little strings on R4 X T2
  - 2d N=(4,4) ADHM GLSM for N=(1,1) little strings

- 6d N=(2,0) Little strings
  - NS5-branes in IIA string theory in 10d flat space
  - T-duality between N=(1,1) and N=(2,0) LSTs
  - 2d N=(0,4) $\hat{A}_{N-1}$ quiver gauge theory for N=(2,0) little strings

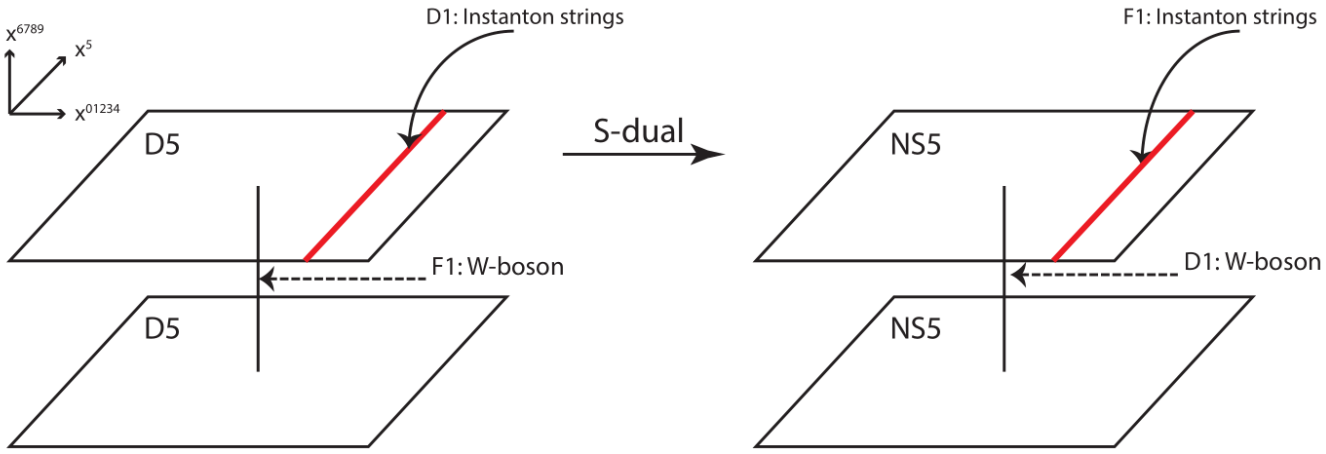
- T-duality
  - T-duality between rank N theories
  - T-duality between rank 1 theories

6d N= (1,1) Little string theory

- NS5-branes in IIB string theory in 10d flat space
  - World-volume theory : 6d N=(1,1) Maximally supersymmetric Yang-Mills.
    - NS5-branes : S-dual of “D5-branes” with the gauge coupling

$$g^2_{\mathrm{YM,D5}} = \frac{1}{T_{\mathrm{D5}}(2\pi\alpha')^2} = (2\pi)^3\alpha' g_s \xrightarrow{\mathrm{S-dual}} g^2_{\mathrm{YM,NS5}} = \frac{1}{T_{\mathrm{NS5}}(2\pi\alpha')^2 g_s} = (2\pi)^3\alpha'$$

    - BPS Degrees of freedom
      - In Coulomb branch , D1-branes : W-bosons
      - Fundamental strings form BPS bound state on NS5s’ : Instanton strings
    - Instanton action : $\frac{4\pi^2}{g^2_{\mathrm{YM,NS5}}} = \frac{1}{2\pi\alpha'} = T_{\mathrm{F1}}$: Tension of the F1.



6d N= (1,1) Little string theory

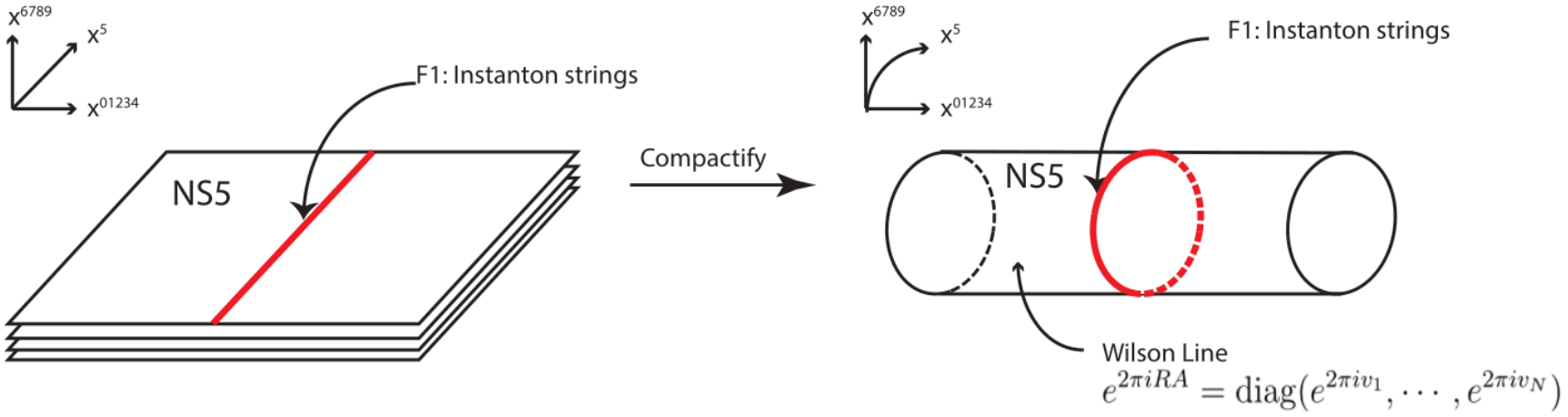
- Little string theory limit
  - The gauge coupling of the NS5-branes : g_s – independent
  - $g_s \to 0, \quad \alpha' \to \text{finite}, \quad E \gtrsim 1/\sqrt{\alpha'}$ limit
    - The gravitational modes are decoupled
    - The perturbative field theory description is totally broken.
    - At this energy scale, Strongly coupled 6d non-gravitational string theory
      → "6d N=(1,1) Little string theory"
    - Little strings : Fundamental strings bound to IIB NS5-branes

6d N= (1,1) Little string theory on $R^{1,4}$ x S^1

- 6d N=(1,1) Little string theory on circle
  - Motivation : T-dual to 6d N=(2,0) Little string theory of IIA NS5-branes.
    - T-Duality : KK-momentum $\longleftrightarrow$ Winding number , with $R \to \alpha'/R$
  - ∃ Wilson line along the circle : $e^{2\pi i R A} = \mathrm{diag}(e^{2\pi i v_1}, \cdots, e^{2\pi i v_N})$
    - KK-Momenta along the circle are fractionalized :

$$P_5 - eA_5 = n - \frac{v_i - v_j}{R_{\mathrm{IIB}}} \quad , \quad (n \in \mathbb{Z})$$

    - Form the charge space of $\hat{A}_{N-1}$ root space.



6d N= (1,1) Little string theory on R^4 x T^2

- Supersymmetric Index for 6d N=(1,1) LST on R^4 X T^2

$$Z(v_i, \epsilon_\pm, q_1, q_2) = \mathrm{Tr}\left[(-1)^F w^W q^{H_L} \bar{q}^{H_R} e^{2\pi i v_i \Pi_i} e^{2\pi i \epsilon_-(2J_{1L})} e^{2\pi i m(2J_{2L})} e^{2\pi i \epsilon_+(2J_{1R}+2J_{2R})}\right]$$

$$\epsilon_\pm = \frac{\epsilon_1 \pm \epsilon_2}{2}$$

  - Global symmetry : $SO(4)_1 \times U(1) \times SO(4)_2$
    - Isometry on NS5-branes' worldvolume : $SO(4)_1 \times U(1) = SU(2)_1^L \times SU(2)_1^R \times U(1)$
    - Isometry on NS5-branes' transverse direction : $SO(4)_2 = SU(2)_2^L \times SU(2)_2^R$
    - w : Winding number of little Strings
    - v_i : Wilson line expectation value → "counting fractional momentum"
  - Result

$$Z_{\mathrm{IIB}}(v_i, \epsilon_\pm, m; q, w) = Z^{\mathrm{IIB}}_{\mathrm{pert}}(v_i, \epsilon_\pm, m; q) Z^{\mathrm{IIB}}_{\mathrm{inst}}(v_i, \epsilon_\pm, m; q, w)$$

$$Z^{\mathrm{IIB}}_{\mathrm{pert}}(v_i, \epsilon_\pm, m; q) = PE\left[\frac{1}{2} I_+ \sum_{i\neq j}^{N} e^{2\pi i(v_i - v_j)} + \left(I_- + (N-1)I_+ + I_+ \sum_{i\neq j}^{N} e^{2\pi i(v_i - v_j)}\right)\frac{q}{1-q}\right]$$

$$Z^{\mathrm{IIB}}_{\mathrm{inst}}(v_i, \epsilon_\pm, m; q, w) = \sum_{k=0}^{\infty} w^k Z_k(v_i, \epsilon_\pm, m; q)$$

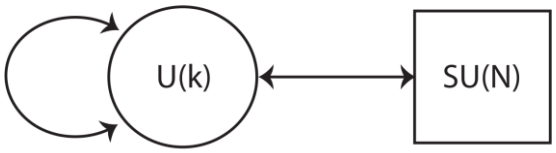
  - $Z_k(v_i, \epsilon_\pm, m; q)$: the elliptic genus of k instanton strings

$$PE\left[f(\epsilon_\pm, m, v_i, q, w)\right] = \exp\left[\sum_{k=1}^{\infty} \frac{1}{k} f(k\epsilon_\pm, km, kv_i, q^k, w^k)\right.$$

$$I_\pm(\epsilon_\pm, m) = \frac{\sinh\frac{2\pi i(m+\epsilon_\pm)}{2} \sinh\frac{2\pi i(m-\epsilon_\pm)}{2}}{\sinh\frac{2\pi i\epsilon_1}{2} \sinh\frac{2\pi i\epsilon_2}{2}}$$

6d N= (1,1) Little string theory on R^4 x T^2

- Gauge theory description of IIB Little strings
  - Worldsheet description :
    - 2d NLSM with the target space of the instanton moduli space
  - As UV-theory : 2d U(k) N=(4,4) ADHM gauged linear sigma model



$\mathcal{N}=(4,4)$	$\mathcal{N}=(0,4)$	Fields	$U(k)$	$U(N)$
Vector	Vector	$A_\mu, \bar{\lambda}_-^{A\dot{\alpha}}$	adj	1
	Twisted Hyper	$\varphi_{aA}, \ \bar{\lambda}_{a+}^{\dot{\alpha}}$	adj	1
Hyper1	Hyper	$a_{\alpha\dot{\beta}}, \ \lambda_{\alpha+}^{A}$	adj	1
	Fermi	$\lambda_{a\beta-}$	adj	1
Hyper2	Hyper	$q_{\dot{\alpha}}, \ \psi_+^A$	$\bar{\square}$	$\square$
	Fermi	ψ_{a-}	$\bar{\square}$	$\square$

  - The elliptic genus of k IIB little strings

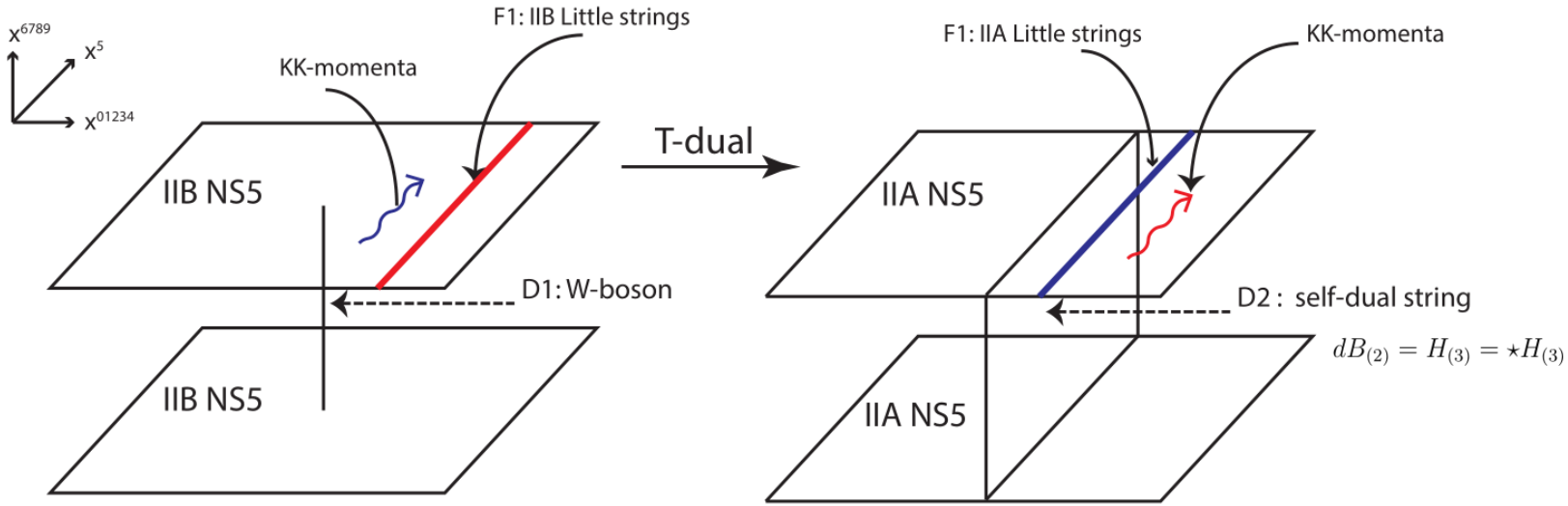
$$Z_k(v_i, \epsilon_\pm, m; q_2) = \sum_{Y:\sum_i |Y_i|=k} \prod_{i,j=1}^{N} \prod_{s\in Y_i} \frac{\theta_1\left(q; E_{ij}+m-\epsilon_-\right)\theta_1\left(q; E_{ij}-m-\epsilon_-\right)}{\theta_1\left(q; E_{ij}-\epsilon_1\right)\theta_1\left(q; E_{ij}+\epsilon_2\right)}$$

$$E_{ij} = v_i - v_j - \epsilon_1 h_i(s) + \epsilon_2 v_j(s)$$

- Computed by Jeffrey-Kirwan residues
  [F.Benini, R.Eager, K. Hori, and Y. Tachikawa]

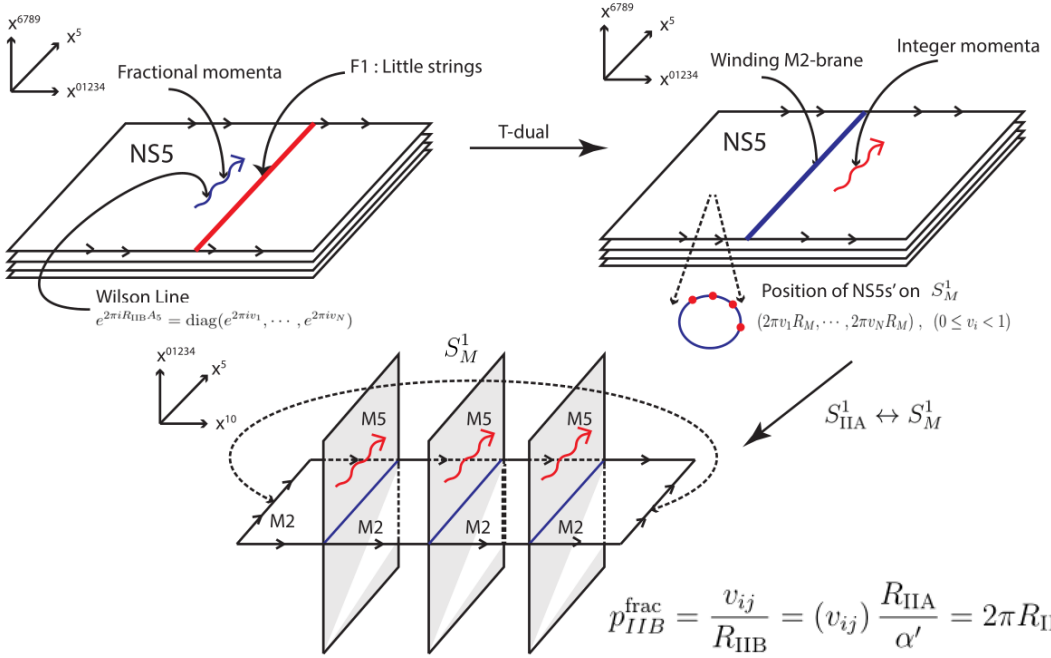
6d N= (2,0) Little string theory

- NS5-branes in IIA string theory
  - World-volume theory :
    - → 6d N=(2,0) Non-abelian tensor multiplet with one compact scalar on S^1_M
      - ∃ Mass scale of $1/R_M$
  - In Tensor branch, Self-dual strings → D2-branes
  - NS5 – F1 BPS bound states
  - $g_s \to 0, \quad \alpha' \to \text{finite}, \quad E \gtrsim 1/\sqrt{\alpha'}$ limit.
    - 6d N=(2,0) Little string theory → T-dual to 6d N=(1,1) Little string theory
  - Little strings : M2-branes wrapping on S^1_M



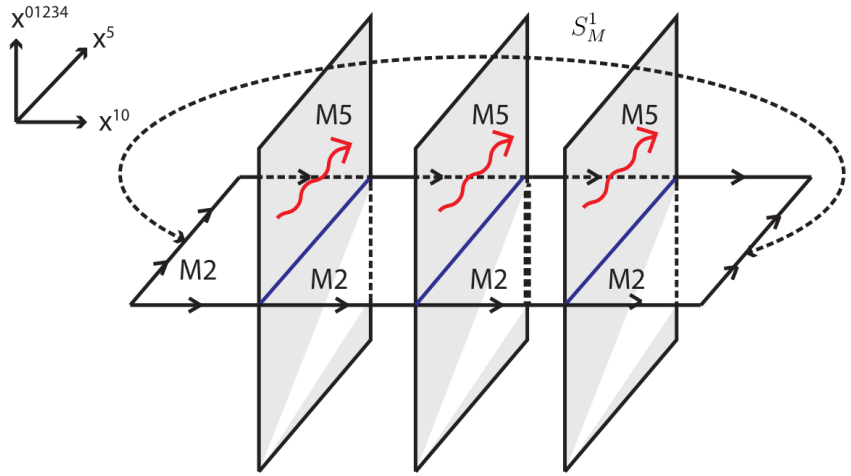
6d N= (2,0) Little string theory on R^4 x T^2

- T-duality between 6d N=(1,1) and N=(2,0) Little strings.
  - Wilson line in 6d N=(1,1) ⟶ Separation of NS5s' along S^1_M
  - Winding # of IIB Little strings ⟶ KK-momentum along $S^1_{\rm IIA}$
  - Fractional momentum ⟶ Fractionalized IIA Little strings
    ~ Winding M2-branes wrapping on $S^1_{IIA} \times I_M(v_{i,i+1})$
- Exchange $S^1_{\rm IIA} \leftrightarrow S^1_M$
  - Obtain M5-branes' circle-compactified abelian tensor branch.
  - Fractional IIA Little strings ⟶ Self-dual strings with $\hat{A}_{N-1}$ type.



$$p^{\rm frac}_{IIB} = \frac{v_{ij}}{R_{\rm IIB}} = (v_{ij})\,\frac{R_{\rm IIA}}{\alpha'} = 2\pi R_{\rm IIA}\frac{v_{ij}}{2\pi\alpha'} = (2\pi v_{ij} R_M)\,(2\pi R_{\rm IIA}) T_{M2} = w^{\rm frac}_{\rm IIA}$$

6d N= (2,0) Little string theory on R^4 x T^2



- Supersymmetric Index for 6d N=(2,0) LST on R^4 X T^2
  - Result

$$Z_{\rm IIA}(v_i,\epsilon_\pm,m;q',w') = Z^{\rm IIA}_{\rm KK}(\epsilon_\pm,m;q')Z^{\rm IIA}_{M2}(v_a,\epsilon_\pm,m;q',w')$$

  - Field theory KK-momentum :

$$Z^{\rm IIA}_{\rm KK}(\epsilon_\pm,m;q') = PE\left[NI_-(\epsilon_\pm,m)\frac{q'}{1-q'}\right]$$

  - Fractional IIA Little strings ⟶ M2-branes' contribution :

$$I_\pm(\epsilon_\pm,m) = \frac{\sinh\frac{2\pi i(m+\epsilon_\pm)}{2}\sinh\frac{2\pi i(m-\epsilon_\pm)}{2}}{\sinh\frac{2\pi i\epsilon_1}{2}\sinh\frac{2\pi i\epsilon_2}{2}}$$

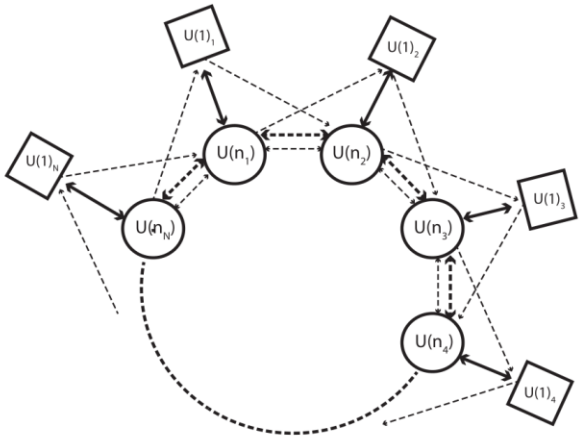
$$Z^{\rm IIA}_{M2}(v_a,\epsilon_\pm,m;q',w') = \sum_{n_i=0}^{\infty} e^{2\pi i\sum_{a=1}^N n_i v_{a,a+1}} Z^{(n_1,\ldots,n_N)}_{M2}(\epsilon_\pm,m;q')$$

6d N= (1,1) Little string theory on R^4 x T^2

- Fractional IIA Little strings
  - Self-dual strings on the circle compactified abelian tensor branch
  - Self-dual strings charged under $\hat{A}_{N-1}$

    [B.Haghighat, A. Iqbal, C. Kozcaz, G. Lockhart, C. Vafa] → A_{N-1} case
  - 2d N=(0,4) circular $\hat{A}_{N-1}$ quiver gauge theory

Multiplet	Fields	$U(n_i)$	$U(1)_i$
Vector	$A_\mu, \bar{\lambda}_-^{A\dot{\alpha}}$	adj	1
Hyper	$q_{\dot{\alpha}},\ \psi_+^A$	$\square_i$	$\square_i$
Hyper	$a_{\alpha\dot{\beta}},\ \lambda_{\alpha+}^A$	adj	1
Twisted Hyper	$\varphi_{aA},\ \bar{\lambda}_{a+}^{\dot{\alpha}}$	$(\square_i, \bar{\square}_{i+1})$	1
Fermi	$\lambda_{a\beta-}$	$(\square_i, \bar{\square}_{i+1})$	1
Fermi	ψ_{1a-}	$\square_i$	$\square_{i-1}$
Fermi	ψ_{2a-}	$\square_i$	$\square_{i+1}$



  - The Elliptic genus of IIA Little strings

$$Z_{M2}^{\mathrm{IIA}}(v_a, \epsilon_\pm, m; q', w') = \sum_{n_i=0}^{\infty} e^{2\pi i \sum_{a=1}^{N} n_i v_{a,a+1}} Z_{M2}^{(n_1,\ldots,n_N)}(\epsilon_\pm, m; q')$$

$$e^{-2\pi i (v_{N+1})} = e^{-2\pi i v_1} w'$$

$$Z_{M2}^{(n_1,\ldots,n_N)}(\epsilon_\pm, m; q') = \sum_{\{Y_1,\cdots,Y_N\};|Y_i|=n_i} \prod_{a=1}^{N} \prod_{s\in Y_i} \frac{\theta_1(q'; E_{ij}^{(a,a+1)} - m + \epsilon_-)\theta_1(q'; E_{ij}^{(a,a-1)} + m + \epsilon_-)}{\theta_1(q'; E_{ij}^{(a,a)} + \epsilon_1)\theta_1(q'; E_{ij}^{(a,a)} - \epsilon_2)}$$

$$E_{ij}^{(a,b)} = (Y_{a,i} - j)\epsilon_1 - (Y_{b,j}^T - i)\epsilon_2$$

T-duality in Rank N Little string theories

- Rank N Little strings
  - 6d N=(1,1) Little string theory

$$Z_{\rm IIB}(v_i,\epsilon_\pm,m;q,w)=Z^{\rm IIB}_{\rm pert}(v_i,\epsilon_\pm,m;q)Z^{\rm IIB}_{\rm inst}(v_i,\epsilon_\pm,m;q,w)$$

$$Z^{\rm IIB}_{\rm pert}(v_i,\epsilon_\pm,m;q)=PE\left[\frac{1}{2}I_+\sum_{i\neq i}^{N}e^{2\pi i(v_i-v_j)}+\left(I_-+(N-1)I_++I_+\sum_{i\neq j}^{N}e^{2\pi i(v_i-v_j)}\right)\frac{q}{1-q}\right]$$

$$Z^{\rm IIB}_{\rm inst}(v_i,\epsilon_\pm,m;q,w)=\sum_{k=0}^{\infty}w^kZ_k(v_i,\epsilon_\pm,m;q)$$

  - 6d N=(2,0) Little string theory

$$Z_{\rm IIA}(v_i,\epsilon_\pm,m;q',w')=Z^{\rm IIA}_{\rm KK}(\epsilon_\pm,m;q')Z^{\rm IIA}_{M2}(v_a,\epsilon_\pm,m;q',w')$$

$$Z^{\rm IIA}_{\rm KK}(\epsilon_\pm,m;q')=PE\left[NI_-(\epsilon_\pm,m)\frac{q'}{1-q'}\right]$$

$$Z^{\rm IIA}_{M2}(v_a,\epsilon_\pm,m;q',w')=\sum_{n_i=0}^{\infty}e^{2\pi i\sum_{a=1}^{N}n_iv_{a,a+1}}Z^{(n_1,\ldots,n_N)}_{M2}(\epsilon_\pm,m;q')$$

$$e^{-2\pi i(v_{N+1})}=e^{-2\pi iv_1}w'$$

- T-duality
  - T-duality $q\leftrightarrow w',\ w\leftrightarrow q'$:
    - Checked to highly nontrivial order of $q,\ w,\ e^{2\pi iv_i}$ in SU(2) gauge theory, [in progress]

T-duality for Rank 1 Little string theories

- Rank 1 Little strings
  - 6d N=(1,1) Little string theory

$$Z_{U(1)\text{IIB}}(\epsilon_\pm, m; q, w) = PE\left[I_-(\epsilon_\pm, m)\frac{q}{1-q}\right]\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}w^n\sum_{\substack{ad=n\\ a,d\in\mathbb{Z}}}\sum_{b(\text{mod } d)} Z_{1-\text{inst}}\left(a\epsilon_\pm, am; \frac{a\tau+b}{d}\right)\right]_{q=e^{2\pi i\tau}}$$

$$Z^{\text{IIB}}_{U(1)\text{ pert}}(\epsilon_\pm, m; q) = PE\left[I_-(\epsilon_\pm, m)\frac{q}{1-q}\right] \qquad Z_{1-\text{inst}}(\epsilon_\pm, m; q) = \frac{\theta_1(q; m+\epsilon_-)\,\theta_1(q; m-\epsilon_-)}{\theta_1(q;\epsilon_1)\,\theta_1(q;\epsilon_2)}$$

  - 6d N=(2,0) Little string theory

$$Z_{U(1)\text{IIA}}(\epsilon_\pm, m; q', w') = PE\left[I_-(\epsilon_\pm, m)\frac{q'}{1-q'}\right]\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}w'^n\sum_{\substack{ad=n\\ a,d\in\mathbb{Z}}}\sum_{b(\text{mod } d)} Z_{\text{M2}}\left(a\epsilon_\pm, am; \frac{a\tau'+b}{d}\right)\right]_{q'=e^{2\pi i\tau'}}$$

$$Z^{\text{IIA}}_{U(1)\text{ KK}}(\epsilon_\pm, m; q') = PE\left[I_-(\epsilon_\pm, m)\frac{q'}{1-q'}\right] \qquad Z_{\text{M2}}(\epsilon_\pm, m; q') = \frac{\theta_1(q'; m+\epsilon_-)\,\theta_1(q'; m-\epsilon_-)}{\theta_1(q';\epsilon_1)\,\theta_1(q';\epsilon_2)}$$

- T-duality
  - T-duality $q \leftrightarrow w',\ w \leftrightarrow q'$:
    - Checked to highly nontrivial order of $q,\ w$
  - Self-T-duality $q \leftrightarrow w,\ q' \leftrightarrow w'$ and Extension of T-duality $q \leftrightarrow q',\ w \leftrightarrow w'$
    - 2d N=(0,4) quiver gauge theory for rank 1 IIA Little strings → Enhanced N=(4,4)
    - GLSM for IIA Little strings = GLSM for IIB Little strings
    - T-duality is extended to Sp(4,Z) duality group , [T.J.Hollowood, A. Iqbal, C. Vafa]

Summary

- Type II Little string theories share T-duality
  exchanging KK-momentum and Winding strings
  with all fractional quantities.

- There are useful 2d gauge theory descriptions
  of Little strings' world-sheet.

- For the rank 1 little string theories,
  T-duality is extended to larger duality symmetry group.